(6 pages) **Reg. No. :** ....

Code No. : 30578 E **Sub. Code : SMMA 61**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2020.

Sixth Semester

Mathematics – Core

COMPLEX ANALYSIS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. If $f(z) = z^2$, then the value of $u(x, y)$

   (a) $x^2 + y^2$ (b) $x^2 - y^2$

   (c) $2xy$ (d) $x + iy$

2. $f(z) = z \operatorname{Im} z$ is differentiable at ___________.

   (a) z (b) 0

   (c) all point (d) none

3. If $\int_C f(z)dz$ where $f(z)=\frac{1}{z}$ and C is the circle $|z|=r$, then $\int_C f(z)dz =$ ___________.

(a) 0 (b) $2\pi i$

(c) $-2\pi i$ (d) $4\pi i$

4. If f a function which is analytic at all points inside and on a simple closed curve C, then $\int_C f(z)dz =$ ___________.

(a) $2\pi i$ (b) πi

(c) 0 (d) $-2\pi i$

5. $1+\frac{z}{1!}+\frac{z^2}{2!}+\ldots\ldots+\frac{z^n}{n!}+\ldots=?$

(a) e^{-z} (b) e^{z}

(c) $\frac{1}{z}$ (d) $-\frac{1}{z}$

6. The poles of $f(z)=\frac{z+1}{z^2-2z}$

(a) 0, 2 (b) 0, 1

(c) 0, –1 (d) 0, –2

7. If $f(z) = \dfrac{-2i}{z^2 + 4z + 1}$, then $\text{Res}\left\{f(z); -2+\sqrt{3}\right\} = ?$

(a) $\dfrac{i}{\sqrt{3}}$ (b) $\dfrac{-i}{\sqrt{3}}$

(c) $\dfrac{2\pi}{\sqrt{3}}$ (d) $\dfrac{-2\pi}{\sqrt{3}}$

8. The value of $\displaystyle\int_{|z|=1} \frac{dz}{z^2 e^z}$

(a) $2\pi i$ (b) $-2\pi i$

(c) πi (d) $-\pi i$

9. The fixed points of the transformation $w = \dfrac{1+z}{1-z}$

(a) $i, -i$ (b) $0, i$

(c) $0, -i$ (d) $i, 2i$

10. A bilinear transformation with only one finite fixed point is called ___________.

(a) parabolic (b) hyperbolic

(c) straight line (d) elliptic

PART B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Verify CR equations for the function $f(z) = z^3$.

Or

(b) Prove that an analytic function in a region with constant modulus is constant.

12. (a) Prove that $\left|\int_a^b f(t)\,dt\right| \leq \int_a^b |f(t)|\,dt$.

Or

(b) Evaluate $\int_C \frac{e^z}{z^2 + 4}\,dz$ where C is positively oriented circle $|z - i| = 2$.

13. (a) Find the Taylor's series expansion for $f(z) = \frac{1}{z}$ about $z = 1$.

Or

(b) Find the residue of $\frac{ze^z}{(z-1)^3}$ at its pole.

14. (a) Evaluate $\int_C \frac{dz}{2z+3}$ where C is $|z| = 2$.

Or

(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$.

15. (a) Find the bilinear transformation which maps the points $z_1 = 2, z_2 = i, z_3 = -2$, onto $w_1 = 1, w_2 = i, w_3 = -1$ respectively.

Or

(b) Find the fixed points of $w = \frac{1}{z-2i}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions.

16. (a) Show that $f(z) = \sqrt{r}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)$ where $r > 0$ and $0 < \theta < 2\pi$ is differentiable and find $f'(z)$.

Or

(a) If $f(z)$ is analytic. Prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left|f(z)^2\right|^2 = 4\left|f'(z)\right|^2$.

17. (a) State and prove Cauchy's theorem.

Or

(b) State and prove Morera's theorem.

18. (a) State and prove Laurent's theorem.

Or

(b) State and prove Cauchy's residue theorem.

19. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{13 + 5\sin\theta}$.

Or

(b) Prove that $\int_0^{\infty} \frac{\cos x}{1 + n^2} dx = \frac{\pi}{2e}$.

20. (a) Find the points where the following mappings are conformal. Also find the critical points of any.

(i) $w = z + \frac{1}{z}$

(ii) $w = e^z$.

Or

(b) Prove that any bilinear transformation preserves cross ratio.

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